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Mathematical Model of Solid Waste Management in Nyamira Municipality Using Fuzzy Goal Programming

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Abstract:
Waste in urban areas is growing rapidly everywhere in the world. Effective methods to address the challenges of solid waste management are critical in achieving a clean and healthy environment. The problem of waste growth in urban areas has been brought about by low collection of waste, illegal and uncontrolled dumping sites and the absence of sewage lines. In Kenya, for instance, the growth of the economy, which resulted in the development of cities and the emergence of towns, led to waste management challenges. Efforts have been directed towards addressing the problem of waste management by various county governments. This study sought to address solid waste disposal and management challenges in Nyamira Municipality. The methods that are being employed to address this challenge are costly and have not yielded desirable results, as was evident from the scattered waste in the streets of Nyamira Municipality. Currently, the management does not carry out any waste recycling and has hired one landfill, which is paid for monthly. This Study formulated a solid waste management tool that involved the construction of a fuzzy goal programming model. The model was solved analytically using the simplex method. A sensitivity analysis was carried out, and graphs were drawn using MATLAB software. The utility of the model was tested using data from Nyamira municipality. The research advocated for waste recycling as one of the ways of managing waste to earn revenue from this recycled waste. The findings of this study are useful in formulating policies such as setting up waste management projects and establishing recycling industries. This will go a long way by reducing the cost by at least 36%. Furthermore, the findings form the basis for future research in related fields.

Keywords: *Waste, fuzzy goal programming, recycling, optimization, simplex*

1. Introduction

1.1. Nyamira Municipality

Nyamira municipality is located in Nyamira County in the former Nyanza province of Kenya. It is the largest town in Nyamira County and its headquarters, with a population of 24483 persons (KNBS 2019 census). The municipality is located in the township ward, Nyamatuta Chache Location. The Municipality consists of Township and Siamai Sub-locations. Other areas contributing to Nyamira Municipal waste include Bigege Sub-location, with a population of 10,579 persons, and Ikobe Sub-location, with a population of 7,882 persons (KNBS 2019). Nyamira Municipality and its periphery generated a lot of solid waste, which was being poorly managed. Solid waste generation in the municipality was from households, schools, markets and hospitals. The main waste management was through collection and dumping in designated dumpsites.

In Nyamira, the rise in population growth, the development of the town, the escalation, and the establishment of more commercial and service activities have led to the production of a lot of solid waste in the town. As a result, there are many problems, such as sewerage blockages and environmental.

Figure 1: Waste at Township Source

Figure 2: Waste at Kemasare Landfill

1.2. Municipal Solid Waste (MSW)

Municipal Solid Waste is an unavoidable by-product of human activity that comprises all wastes produced within the territory of the municipality. It may also be referred to as waste produced, gathered, transported and discarded within the boundaries of the municipality.

According to Hoornweg *et al.* (2005), Municipal Solid Waste is described as waste gathered and discarded by the authority of the municipal at the municipal dumpsites and includes wastes from residences, industries, institutions, commercial centers and construction and demolition sites.

In many cases, municipal solid waste includes food remnants, garbage from residences, and street sweepings.

1.3. Solid Waste Management (SWM)

Solid Waste Management is an expression that refers to the task of gathering, treating and dumping solid waste.

Solid Waste Management (SWM) is also described as a set of systematic and consistent rules relating to the control of generating, keeping, collecting, transporting, procession of waste and waste land-filling to the best principles of public health, conservation of resources, aesthetics, economy, other requirements of the environment and what the public requires.

Pattnaik and Reddy (2010) described Solid waste management as the effective and efficient collection, transporting, and disposal of waste from residences, street sweepings, construction sites, non-dangerous industries, and imports, including secondhand clothes popularly known as *mitumba* in Kenya.

Solid waste management means a sequence of activities covering all functional elements, including waste generation, handling, separating, storing, and processing at the generation point, collection, separating, processing, and transformation at treatment facilities, and final disposal.

1.4. Mathematical Modeling

Mathematical modeling is the process of developing mathematical models. In mathematics, modeling involves converting real-world problems, such as solid waste management, into mathematical problems that can be solved using equations and mathematical symbols. Mathematical modeling can also be described as converting real problems into mathematical forms.

Galbraith and Clatworthy (1990) described mathematical modeling as the mathematical application of finding solutions for problems that are not structured in real-life situations. In modeling, mathematical approaches are used to solve challenges related to problems in real-life situations. Real-life problems that we come across are converted into mathematical problems and solved using mathematical techniques (Cheng *et al*., 2001).

According to Sarakikya (2020), the mathematical model consists of governing equations, assumptions and constraints, and initial and boundary conditions. Various classifications of conditions can be used for mathematical models depending on their structure. It is significant to derive equations so that their differences and similarities are pointed out and reflected on for possible implications in their implementation to mathematical modeling. The methods of Mathematical modeling produce a virtual reality which when applied, may populate with everything that moves, irrespective of scale. Mathematical modeling allows the user to carry out experiments that, in real life, are difficult, expensive and, dangerous or impossible to measure.

1.5. Fuzzy Set

Fuzzy sets are sets with a degree of membership for their elements. A fuzzy set is an ordered pair (X, m) where X is a non-empty set known as the universe of discourse and m is a mapping $m : X \to [0,1]$ (Zadeh L A 1965, Bushra Hussien Aliwi 2009)

For each $x \in X$ the value of $m\,(x)$ the degree of membership of $x \in (X, m)$ where:

$m = \mu_A(x)$ is a membership function of the fuzzy set A defined by:

$$\mu_A(x) = \begin{cases} 1\ if\ x \in A \\ 0\ if\ x \notin A \end{cases} \qquad (1)$$

Therefore,

$$\mu_A(x) \in [0,1] \qquad (2)$$

For all $x \in X$, then

i) x is not inclusive in the fuzzy set A if $m(x) = 0$

ii) x is partially inclusive in the fuzzy set A if $0 < m(x) < 1$

iii) x is fully inclusive in the fuzzy set A if $m(x) = 1$

1.6. Triangular Membership Function

There are various membership functions, including triangular, trapezoidal, piecewise, etc. The triangular membership function is distinguished by three variables $\{a, b, c\}$ where a, b and c represent the coordinates of x for the three vertices in a fuzzy set A of $\mu_A(x)$ (a is the lowest limit and c is the upper limit where membership degree is zero and b is the center where membership degree is 1). Therefore:

$$\mu_A(x) = \begin{Bmatrix} 0 & if\ x \le a \\ \frac{x-a}{b-a} & if\ a \le x \le b \\ \frac{c-x}{c-b} & \text{if b} \le \text{x} \le \text{c} \\ 0 & if\ \ c \le x \end{Bmatrix} \qquad (3)$$

1.7. Goal Programming

Goal programming is a branch of multi-objective optimization. It involves multi-criteria decision analysis (MCDA) (Watada J *et al*. 2022). It is a generalization of linear programming in the sense that it can handle multiple variables that are conflicting in nature. It can be used to solve the conflicting aspiration levels in terms of minimizing cost and maximizing profit. The deviations in the target of achievement defined in terms of minimizing cost and maximizing profit are calculated. The deviation from the set targets of achievement is then minimized to the satisfaction of the decision-maker(s). Goal programming was used first by Cooper, Charnes and Ferguson in 1955. The following three types of analysis are performed using goal programming:

- Determine the number of resources needed to achieve the desired set of objectives.
- Determine the attainment degree of the set objectives using the resources available.
- Provide the best solution under different circumstances, such as shifting priorities of the goals and changing the amount of resources.

The overall objective of GP is to minimize deviations that arise among the levels of attainment of goals and their levels of aspiration. According to Charnes and Cooper (1977), the GP is expressed as follows:

$$\sum_{i=1}^{k} |\mathrm{F_i(x)} - \mathrm{g_i}| \qquad (4)$$

Subject to the following:

$X = \{x \in \mathbb{R}^n\}$

Here, $\mathrm{F_i(x)}$ is the linear functional achievement of the i^{th} goal, and $\mathrm{g_i}$ is the level of aspiration of the i^{th} goal.

Therefore,

$$F_i(x) - g_i = d_i^+ - d_i^-\ for\ d_i^+, d_i^- \ge 0 \qquad (5)$$

Therefore, the GP can be formulated as follows:

$$\sum_{i=1}^{k} |F_i(x) - g_i| = \sum_{i=1}^{k} |\mathrm{d_i^+} - \mathrm{d_i^-}| \qquad (6)$$

Subject to the following conditions:

$$F_i(x) - d_i^+ + d_i^- - g_i = 0_i\ i = 1,2, \dots k \qquad (7)$$

$$X = \{x \in \mathbb{R}^n\} \qquad (8)$$

1.8. Statement of the Problem

As one walks around Nyamira Municipality, scattered solid wastes are seen in the streets due to poor management. These wastes require large amounts of land so that solid waste management projects can be implemented. It was a hard test to get a large amount of land in the municipality of Nyamira, given that the municipality had set aside land that was not even enough for waste disposal and management. This study looked at Solid Waste management and disposal in Nyamira Municipality. The methods that were employed to address those challenges were costly and did not yield desirable results. This study has developed a fuzzy goal programming mathematical model that can be used to manage solid waste in Nyamira Municipality.

1.9. Objectives of the Study

- To formulate a solid waste management FGP model.
- To determine the optimum solution from the model in (i) above using the simplex method.

2. Literature Review

Shaban *et al*. (2022) formulated a mixed integer linear programming (MILP) model for solid waste management that integrated generation of waste, collection of waste, transfer of waste, recycling projects, incinerators and landfills. The model aimed to determine the locations, optimal number of different facilities, and the flow of waste so that the daily cost of the system could be minimized. A case study of the model was implemented in Fayoum, Egypt. The obtained data were solved numerically using LINGO computer software. The results indicated that the optimal design for the management system of solid waste in Fayoum Governorate could yield the optimal solution by installing four centres for collecting waste, one facility for recycling and one landfill strategically located. This showed a linear reduction in the net daily cost as there was an increase in the recycling plant.

Govindan *et al.* (2021) presented a model of a bi-objective mixed integer linear programming (MILP) for the management of medical waste during an outbreak of COVID-19 for taking care of both non-infectious waste and infectious waste in unpredictable environments. The objectives of the model simultaneously were to minimize the overall cost and population exposure risk to pollution. As a result, an FGP model was designed to get the solution for the created model. The data used in the study was obtained from Tehran Municipality. The factors such as separating non-infectious waste from infectious waste were considered during the stages of waste collection by vehicles, reducing the waiting time for the vehicles entering waste production centers and failure for the vehicle to carry infectious waste. The obtained results were useful to managers and decision-makers. For instance, the results indicated that vehicles with a low possibility of failure should be allocated to collect hazardous waste, and those with a higher probability should be assigned to collect non-hazardous wastes. The model was found to be effective and efficient since the waiting time for the vehicles entering to carry infectious waste in the waste production centers decreased to zero. The model, which is practical and flexible, was presented for the management of medical waste during the pandemic of COVID 19. The findings were helpful to the decision-makers and managers for the scenario to be adopted that imposes the least exposure risk to the population.

Mehdi *et al*. (2021) developed a bi-objective optimization model that aimed to minimize the cost of the location of the facility, transportation organization, and the emissions of pollutants from the environment. The uncertainty of the problem and the quantity of waste generated as a random parameter were considered. As a result, a stochastic mathematical programming model with probable constraints was created. The results revealed that increasing the capacity levels would lead to a decrease in the Cost of the location of the facility, transportation, organization and the emissions of environmental pollutants.

Onchong'a *et al*. (2019) conducted a study on the effect of participation of the stakeholders in the management of solid waste implementation Projects in the county of Nyamira, Kenya. A multiple regression analysis was adopted to establish the relationship between independent and dependent parameters. It was discovered that there existed an impressive relationship between the participation of stakeholders and the management of solid waste project implementation in Nyamira County. The results showed that the involvement of stakeholders was significant. From the basis of the outcome, it was proposed that the county is required to continue allowing stakeholders to be involved in the solid waste management implementation projects. The identified stakeholders should be involved in the early phases of SWM projects. This is to ensure that their interests and concerns are captured, addressed and incorporated into SWM Implementation projects. The involvement of stakeholders was also found to be relevant since it offered assistance in monitoring and evaluating during the implementation of these solid waste management projects.

Kalu *et al.* (2017) worked on a mathematical model for the SWM system in the Nigeria Aba Metropolis municipality of Abia state. In the study, it was observed that the minimum cost of waste management decreases with the increase in the capacities of the collection centers. The research indicated that designing the centers of waste collection with maximum capacities minimizes the cost, provided that other factors are held constant.

Soltani *et al*. (2015) explored multi-criteria decision-making for managing solid waste in the municipality involving numerous stakeholders. The study considered MSWM to be a complex process that comprises economic, social and environmental criteria. Besides, it provides a process of making decisions in the management of solid waste challenges in the municipality, such as finding appropriate disposal sites for solid waste and coming up with strategies that will involve as many stakeholders as possible, including municipalities, national government, experts, industries and even the public. The research showed that the analytical hierarchy process is the most effective approach involving various stakeholders in the management of solid waste in municipalities.

Yu *et al. (*2015) developed a linear and dynamic bi-targeted programming mathematical model for optimizing the long-term performance of the management of solid waste in the municipality. The model proposed deals simultaneously with the productivity of economic calculation and pollution of the environment within various time periods for municipal SWM. The study carried out the optimal change across the horizon entirely, which indicated the accuracy of the model that was developed. The established mathematical model was solved by LINGO Software, and the provided solution was found to be effective in long-term planning operations for the system of solid waste in the municipality.

Arena and Di Gregorio (2014) investigated the management planning of municipal solid waste based on the analysis of the movement of the waste. The outcome from the management systems of solid waste in the municipal was described in the study, and it was discovered that combining the materials and substances movement analysis with environmental assessment methods is a toolbox that is effective for comparing the scenario and management of solid waste. In the paper, a mathematical model for optimizing the current system of management of solid waste in Tehran and the identification of the proper number of facilities for waste transfer and waste processing was developed. The model proposal provides an alternative way of improving the SWM system by reducing the number of transfer stations and units for waste processing.

Lohri *et al.* (2014) analyzed the sustainability of finance in the management of solid waste in the Municipal system costs and revenues in Ethiopia, the city of Bahir Dar. The research examined the data by analyzing the cost-income from July the year 2009 up to 2011 in June. From the analysis, it was noted that the system of the total costs of management of solid waste in Bahir Dar had dramatically increased within the considered period due to increasing costs relating to the transportation of waste, such as the cost of collecting waste from the residential, companies, commercial centers and institutions such as stadiums, schools, hospitals, among others. The results obtained from the research indicated that the existence of the correct analysis structure of costs and revenue from the system of Solid Waste Management increased productivity and income in relation to the cost. The results also showed that the cooperation between the private sector and the municipality is a sufficient solution for improving the sustainability of the financial system of Solid Waste Management.

Srivastava and Nema (2011) developed a mathematical fuzzy parametric programming model for the identification of treatment facilities and facilities for disposal for management of solid waste capacities, planning and waste flow allocation under uncertainty. A deterministic waste allocation scheme was generated by the formulated model but with no basic provision to support the generation of many decision options, desired treatment, facilities for disposal, SWM planning capacities, and waste allocation movement in an uncertain environment. The model proposed generated waste allocation deterministic schemes without proving bases for the support of generating many decision options corresponding to the conditions of uncertain systems.

Li *et al.* (2008) explored the uncertainties related to solid waste management operational costs and transportation costs. The study formulated an inexact stochastic quadratic programming model that was expressed in possibility distribution terms and discrete intervals.

2.1. Conceptual Framework

A solid waste management system involves source, sorting, waste recycling, market for waste recycling, and disposal. Sorting was done at the sources, and after sorting, waste to be recycled was taken to a recycling facility where transportation and operating costs were incurred, and revenue was generated from the sales of recycled waste. Non-recyclable waste is taken to the landfill, where transport costs are incurred.

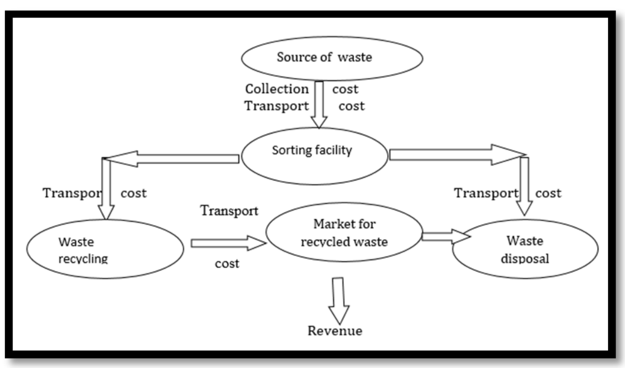

Figure 3: Conceptual Framework of the Model Showing Components of SWM System

3. Methodology

3.1. Indices

The model was formulated by the following indices:
i represents the source of waste; $i = 1,2 \dots n$
j represents disposal and recycling facilities; $j = 1,2$.
Here $j = 1$ indicates landfill and $j = 2$ represents recycling facilities.
WTC_{ij} represents the cost of waste transportation from source i to facility j.
OC_2 represents the cost of operation of the recycling facility.
LC represents landfill capacity.
RE_2 represents revenue being generated from the recycling facility
RC represents the capacity of the recycling facility
W_{ij} represents waste disposal demand
Decision variables
X_{ij}: Quantity of waste from source i to facility j.

3.2. Formulation of the Model

Using the notations defined above, the mathematical model for multi-objective solid waste management (MOSWM) in the case of the deterministic parameters is formulated as:

Minimize $Z_1 = \sum_{i=1}^{n} WTC_{i1} X_{ij} + \sum_{i=1}^{n} WTC_{i2}\, X_{ij} + \sum_{i=1}^{n}\ OC_2\, X_{ij}$ (9)

Maximize $Z_2 = \sum_{i=1}^{n} RE_2\ X_{ij}$ (10)

Subject to the following constraints:

i) Waste Disposal Demand Constraint

$\sum_{i=1}^{n} \sum_{j=1}^{2} X_{ij} \geq W_{ij}$ i = 1,2 … n, (j = 1,2) (11)

ii) Landfill Capacity Constraint

$\sum_{i=1}^{n} X_{1i} \leq LC$ (12)

iii) Recycling Facility Capacity Constraints

$\sum_{i=1}^{n} X_{ij} \leq RC$

iv) Non-negativity constraint

$X_{ij} \gtrsim 0$ (13)

In the formulation of MOSWM model, the parameters are assumed to take deterministic values. However, in most of practical situations, these may take imprecise values due to the reasons listed below:

- The capacity of landfill may vary, especially when the model is developed over a long period of time.
- The generation of waste in Nyamira Municipality is uncertain to the decision-maker.
- The price of recycled items may depend on the decision-maker.

Such vagueness in the information is critical and cannot be captured in deterministic problems. Thus, the optimal results obtained from deterministic formulations may not sufficiently serve the actual purpose of modeling the problem. Because of this, the study considers the model with imprecise information.

In light of the above discussion regarding the MOSWM model, the fuzzy formulation of the problem is done by replacing all deterministic parameters WTC_{ij}, OC_2 and RE_2 with fuzzy parameters $\widetilde{WTC}_{ij}$, $\widetilde{OC}_2$ and $\widetilde{RE}_2$ and is, therefore, expressed as:

Minimize $\tilde{z}_1 \cong \sum_{i=1}^{n} \widetilde{WTC}_{i1}\, X_{ij} + \sum_{i=1}^{n} \widetilde{WTC}_{i2} X_{ij} + \sum_{i=1}^{n} \widetilde{OC}_2 X_{ij}$ (14)

Maximize $\tilde{z}_2 \cong \sum_{i=1}^{n} \widetilde{RE}_2\, X_{ij}$ (15)

Subject to the following constraints:

$\sum_{i=1}^{n} \sum_{j=1}^{2} X_{ij} \gtrsim W_{ij}\ i = 1,2 \ldots n, j = 1,2$ (16)

$\sum_{i=1}^{n} X_{1i} \lesssim \widetilde{LC}$ (17)

$\sum_{i=1}^{n} X_{ij} \lesssim \widetilde{RC}$ (18)

$X_{ij} \gtrsim 0$

3.3. Fuzzy Goal Programming Model for Multi-Objective Solid Waste Management

Considering the parameters provided above, decision-makers may have fuzzy goals for each objective. To obtain the aspiration level for the fuzzy goals, each objective is solved individually for the modified set of system constraints defined in the MOSWM model.

Using Zimmermann's (1978) approach, a fuzzy goal MSWM model is expressed as follows:

Minimize $\tilde{z}_1 \cong \sum_{i=1}^{n} \widetilde{WTC}_{i1} X_{ij} + \sum_{i=1}^{n} \widetilde{WTC}_{i2}\, X_{ij} + \sum_{i=1}^{n} \widetilde{OC}_2 X_{ij} \ \lesssim g_1$ (19)

Maximize $\tilde{z}_2 \cong \sum_{i=1}^{n} \widetilde{RE}_2\, X_{ij} \gtrsim g_2$ (20)

Subject to the constraints (16) to (18) and (13)

Where g_1 and g_2 represent aspiration levels for the first and second goals, respectively. It also means that the Decision-Maker (DM) may be satisfied even if it is greater than in the case of the first goal and less than for the second goal up to a certain tolerance limit.

Considering that the generation of waste can be uncertain, so can landfill and recycling facility capacities and budget allocation, they are said to be fuzzy. The model is modified by substitutingW_{ij}, LC and RC_j with d, l and r, respectively, as follows:

Minimize $\tilde{z}_1 \cong \sum_{i=1}^{n} \widetilde{WTC}_{i1} X_{ij} + \sum_{i=1}^{n} \widetilde{WTC}_{i2}\, X_{ij} + \sum_{i=1}^{n} \widetilde{OC}_2 X_{ij} \ \lesssim g_1$

Maximize $\tilde{z}_2 \cong \sum_{i=1}^{n} \widetilde{RE}_2\, X_{ij} \gtrsim g_2$

Subject to the following constraints:

$\sum_{i=1}^{n} \sum_{j=1}^{m} X_{ij} \ \gtrsim d, i = 1,2..n, j = 1,2$ (21)

$\sum_{i=1}^{n} X_{ij} \lesssim \tilde{l}$ (22)

$\sum_{i=1}^{n} X_{ij} \lesssim \tilde{r},$ (23)

$X_{ij} \gtrsim 0$

3.4. Defuzzification

There is a need to denazify the constraints to have corresponding crisp values. The study applies the centroid defuzzification method, also called the centre of the area (COA), which is the most prevalent defuzzification method (Ross, 2004) whose underlying principle is

$$x^* = \frac{\mu(x)xdx}{\mu(x)dx} \quad (24)$$

Where x^* is the defuzzified value, $x = (a, b, c)$ indicates the element in x^*, and $\mu(x)$ is its associated membership function.

Buyukozkan (2012) translated equation (24) while defuzzifying a TFN by taking the $\boldsymbol{\alpha}$-cut.

Set $\widetilde{C_\alpha}$, as follows:

$$x^* = \frac{1}{2}\int_0^1 (\inf \widetilde{C_\alpha} + \sup \widetilde{C_\alpha})\, d\alpha \quad (25)$$

with $\boldsymbol{\alpha}$-cut set

$\widetilde{C_\alpha} = [a + (b-a)\alpha, \mathrm{c} - (\mathrm{c}-\mathrm{b})\alpha$

Equation (3.3.2) is further transformed as:

$$x^* = \frac{1}{2}\int_0^1 [a + (b-a)\alpha + \mathrm{c} - (\mathrm{c}-\mathrm{b})\alpha]\, d\alpha$$

$$= \frac{a+c}{2} + \frac{1}{2}\int_0^1 (2b - a - c)\alpha\, d\alpha (3.3.4)$$

$$= \frac{a+2b+c}{4} \quad (26)$$

Let d, l and r be triangular fuzzy numbers defined by:

$d = (d^1, d^2, d^3)$, $l = (l^1, l^2, l^3)$ and $r = (r^1, r^2, r^3)$ with their membership functions as u_d, u_l and u_r, respectively.

Using the centroid defuzzification method, we obtain their corresponding defuzzified values as:

$$d_c = \frac{d^1 + 2d^2 + d^3}{4} \quad (27)$$

In the same way, we obtain l_c and r_c

The triangular MF is given by:

$$u_d(d_c) = \begin{cases} \frac{d-d^1}{d-d^2} if d^1 \precsim d_c \precsim d^2 \\ \frac{d^3-d}{d^3-d^2} if d^3 \precsim d_c \precsim d^3 \\ 0 \qquad otherwise \end{cases} \quad (28)$$

In the same way, we obtain $u_l(l_c)$ and $u_r(r_c)$

After obtaining the fuzzified values d_c, l_c and $r_c \forall ij$ the model becomes:

Minimize $\widetilde{z_1} \cong \sum_{i=1}^n \widetilde{WTC}_{\iota 1} X_{ij} + \sum_{i=1}^n \widetilde{WTC}_{\iota 2} X_{ij} + \sum_{i=1}^n \widetilde{OC}_2 X_{ij} \precsim g_1$

Maximize $\widetilde{z_2} \cong \sum_{i=1}^n \widetilde{RE}_2 X_{ij} \gtrsim g_2$

Subject to the following constraints:

$\sum_{i=1}^n \sum_{j=1}^m X_{ij} = d_c,\ i = 1,2..n, j = 1,2$ (29)

$\sum_{i=1}^n X_{ij} \le l_c$ j=1 (30)

$\sum_{i=1}^n X_{ij} \le r_c$, j=2…m (31)

$X_{ij} \ge 0$

In GP, since the goals in most cases are conflicting, the decision-maker may not be able to achieve their aspirations, and therefore, deviations may occur, such as 'underachievement' and 'overachievement'. Taking d_i^- to represent under deviation and d_i^+ to represent over deviation, the model above can be expressed as:

Minimize $\widetilde{z_1} \cong \sum_{i=1}^n \widetilde{WTC}_{\iota 1} X_{ij} + \sum_{i=1}^n \widetilde{WTC}_{\iota 2} X_{ij} + \sum_{i=1}^n \widetilde{OC}_2 X_{ij} + d_1^- - d_1^+ = g_1$ (32)

Maximize $\widetilde{z_2} \cong \sum_{i=1}^n \widetilde{RE}_2 X_{ij} + d_2^- - d_2^+ = g_2$ (33)

Subject to the above system of constraints from equations (31) to (33) and (13).

The objective of a GP is to minimize unwanted deviations from objective functions. In this case, for the first objective function, the decision maker may not wish to overspend and, therefore, the need to minimize d_1^+, and in the second objective function, the revenue should not be below the wish of the decision maker; therefore, the need to minimize d_2^-. The objective coefficients $\widetilde{WTC}_{\iota 1}, \widetilde{WTC}_{\iota 2}$, $\widetilde{RE}_2$ and $\widetilde{OC}_2$ are taken as fuzzy numbers, implying that the objectives Z_1 and Z_2 must also be fuzzy numbers. Therefore, all fuzzy numbers are triangular fuzzy numbers of the form $a = (a^L, a, a^U)$, where the superscripts L and U represent lower tolerance and upper tolerance, respectively.

Let $Z_1 \cong [Z_1^L, Z_1^U]$ and $Z_2 \cong [Z_2^L, Z_2^U]$. To minimize the objective function, the lower tolerance corresponds to the aspiration level g_1 while the upper tolerance corresponds to the aspiration level g_2 for maximization of the objective function. Therefore, the model is reformulated as:

Optimize F $= d_1^+ + d_2^-$ (34)

Subject to:

$\sum_{i=1}^n \widetilde{WTC}_{\iota 1} X_{ij} + \sum_{i=1}^n \widetilde{WTC}_{\iota 2} X_{ij} + \sum_{i=1}^n \widetilde{OC}_2 X_{ij} + d_1^- - d_1^+ = g_1$ (35)

$\sum_{i=1}^n \widetilde{RE}_2 X_{i2} + d_2^- - d_2^+ = g_2$ (36)

And the other system of constraints from (31) to (33) and (13).

3.5. Construction of Membership Function for FGP

Fuzzy goals are quantified by eliciting the corresponding membership functions on the basis of the achieved values. Thus, the linear membership function of each of the objective functions is written as:

$$\mu Z_1 \cong \begin{cases} 0 & if Z_1 \gtrsim Z_1^U \\ \frac{Z_1^U - Z_1}{Z_1^U - g_1} & if \quad g_1 \lesssim Z_1 \lesssim Z_1^U \\ 1 & if Z_1 \lesssim g_1 \end{cases} \quad (37)$$

Where Z_1^U is the upper tolerance limit for the first goal, and $Z_1^U - g_1$ is the tolerance which is arbitrarily chosen. Also,

$$\mu Z_2 \cong \begin{cases} 0 & if Z_2 \lesssim Z_2^L \\ \frac{Z_2 - Z_2^L}{g_2 - Z_2^L} & if \quad Z_2^L \lesssim Z_2 \lesssim g_2 \\ 1 & if Z_2 \gtrsim g_2 \end{cases} \quad (38)$$

Where Z_2^L is the lower tolerance limit for the second goal and $g_2 - Z_2^L$ is the tolerance which is also arbitrarily chosen.

In fuzzy programming approaches, the highest possible value of the membership function is 1, while the lowest is 0.

3.6. Solution by Using Simplex Method

The above model was solved using an analytical approach using the simplex method. The simplex method is an analytical method for finding solutions for linear programming models by using pivot variables, tableaus, and slack variables as a way of looking for the desired optimization problem's solution. The following steps are followed to find the solution:

- Check that $x_i \geq 0,\ \forall i = 1,2 \dots n$. If not, then we replace x_i by $-y_i$ in the given problem so that $y_i \geq 0$
- Check if the given problem is maximization. If the problem given is minimization, then we multiply it by -1 in the objective function to convert it to maximization.
- We check that $e_i, b_i, l_i, r_j \geq 0$. If not, then we multiply the corresponding constraints by: -1 so that $e_i, b_i, l_i, r_j \geq 0$.
- Convert all the inequalities of the constraints into equations by introducing fuzzy slack variables.
- The fuzzy variables $x_1, x_2 \dots x_n$ constituting identity sub-matrix given the basis.
- $x_i = (x_1, x_2 \dots x_n)$ is the coefficient matrix. The values of $x_1, x_2 \dots x_n$ can be obtained by putting the values of the remaining (N-m) fuzzy non-basic variables equal to zero.
- Let c_{ij} where $i = 1,2 \dots n, j = 1,2 \dots m$ be coefficients of $x_{11}, x_{22} \dots x_{nm}$, respectively, in the objective functions.
- Construct a fuzzy simplex tableau as follows:

×	x_1	x_2	…	x_n
x_1	x_{11}	x_{12}	…	x_{1n}
x_2	x_{21}	x_{22}	…	x_{2n}
. . .				
x_m	x_{m1}	x_{m2}		x_{mn}
$D = \sum_{i=1}^{n} c_k x_i$	$D_1 = C_1$	$D_2 = C_2$	…	$D_N = C_N$

Table 1: Simplex Algorithm

- Perform iterations

3.7. Case Study

Nyamira municipality generates about 40 tons of solid waste per day (Environmental Department Nyamira municipality). Out of this figure (40 tons per day), only 30 tons per day was being collected, leaving more waste uncollected every day. This is why a heaping amount of waste is seen in the streets of the municipality, as shown in figure 2. Recovery processes in the municipality mainly include plastic, metallic and rubber waste recycling centers. Further, recyclable waste is mostly locally collected by scavengers (*chakras*) and then taken to vendors and recycling/reuse centres.

Currently, there is no landfill facility owned by the municipality. The municipality dumps its waste at a privately owned dumpsite for which it pays a fee of Ksh. 18,000 per month. The municipality owns a tipper truck and a tractor with a capacity of 8 tons and 7 tons, respectively. The vehicles collect waste at designated points every weekday. The municipality has contracted casual workers to operate the vehicles. The tipper truck has 12 casual workers, while the lorry has 8 casual workers and 10 casual workers whose role is to manage the landfill. Each casual is paid Ksh 800 per day if they work for 5 days in a week. In the last financial year, the municipality had the following expenditures, as shown in table 2.

No.	Item	Cost	Total (Ksh)
1	Vehicle maintenance	i) Tipper truck - @150,000 per month × 12 ii) Tractor - @ 100,000 per month × 12	3,000,000
2	Salaries	i) Casuals - @800 × 30 casuals × 20days × 12 months = 5,756,000 ii) Drivers- @ 25,000 × 4 drivers × 12 months =1,200,000 iii) Supervisors - @30,000 × 6 supervisors × 12 months =2,160,000 iv) Other employees in the department- for 450,000 × 12months = 5,400,000	14,520,000
3.	Road maintenance	@ 5,000,000 per year	5,000,000
4.	Fuel	@210 × 33.5 liters per day × 20 days × 12 months	3,027,310
5.	Overall	@7500 × 36	270,000
6	Gumboots	@ 5000 × 36	180,000
7.	Gloves	@ 2500 × 36	90,000
8	Spades	@ 2500 × 10	25,000
9	Wheelbarrows	@ 15000 × 10	150,000
10	Dustbins	@6000 × 70	420,000
		Total	26,682,310

Table 2: Nyamira Municipality Waste Management Expenditure

Currently, there is no recycling facility owned by the municipality. Recycling is being done on a scale in privately owned facilities. These recycling facilities get the waste from sources and other sources from the dump site. Plastic waste was found to be the most recycled/reused solid waste.

3.8. Proposed Fuzzy Goal Programming Mathematical Model

The proposed Fuzzy Goal Programming (FGP) model is a Mathematical Model that optimizes the objectives of the total cost of SWM, which includes the cost of transporting different types of waste and revenue collected from recycled waste. The nodes of the transportation network consist of collection, recycling, and final disposal nodes. The proposed (FGP) mathematical model was formulated to determine the establishment of recycling centers at a minimum cost. The study realized that measuring transportation costs per ton is the most preferred method in most towns in developing countries. With the current situation in Nyamira municipality, where the use of technology to measure waste as it is transported from the waste sources is not available, this study estimated transportation costs in terms of costs per ton of a vehicle from the waste collection center to the landfill and recycling facilities.

3.9. Description of the Conceptual Framework of the Proposed Model for the MSWM System

The main focus of the model is to plan the MSW management by defining the refuse flows that have to be sent to recycling centers or to the final disposal sites, from waste sources (residences, markets, schools, restaurants, institutions, hotels, etc.). All sorts of wastes produced daily will be moved to collection center i at the expense of generators and some fractions of recyclable/reusable waste are bought/collected and directly taken to vendors/recycling/reuse centers by scavengers. Collection centers are the officially known/adapted points where wastes of a different kind from nearby places (waste sources) are dumped, after which they will be loaded/moved to recycling centers other than to the final disposal site (j=1). Recycling/reusing waste material center is the point where recycling recyclable waste materials such as plastics, rubber and metals are technically feasible. The advantages of recycling waste materials are reducing the amount of waste that reaches the final disposal site. The final disposal site is the final destination where the waste residue reaches either directly or after passing through recycling facilities. It utilizes a land area to collect the waste with or without separation. Its advantage is that all waste (except hazardous materials) can be dumped without separation.

In this study, there are five waste source locations, one landfill location and one recycling facility location with different sections for recycling different wastes, that is, for papers, plastics and metals. The names for waste sources, recycling plants, and landfill locations were taken from subdivisions of the municipality. It is important to note that the cost of waste transportation parameter data has been carefully chosen as close to reality as possible in the municipal. The model assumes that all the wastes taken to the recycling facility are recycled, and the buyers for the recycled wastes come for them at the facility. Therefore, management will not incur any more costs.

3.10. Data Used to Test the Model

Table 3 gives locations for sources of waste, recycling plants, and landfills. Table 4 gives waste source locations and the amount of waste in tones at these sources. Table 5 shows the distances of waste sources to the landfill. Table 6 gives distances of waste sources to the recycling facility. Tables 7 and 8 give the capacities of the recycling facility and landfill facility, respectively. Table 10 gives transportation costs to the landfill and recycling facility in TFN. Table 11 gives the recycling facility operation cost and the revenue generated in TFN.

	Node Type	Locations
1	Waste sources	Township, Miruka, Kebirigo, Nyamaiya, Tinga
2	Recycling plants	Township
3	Landfill	Kemasare

Table 3: Locations for Waste Sources, Recycling Facilities and Landfill

	Waste Sources Locations	Distance (Km)
1	Township	12
2	Miruka	8
3	Kebirigo	22
4	Nyamaiya	5
5	Tinga	20

Table 4: Distance in Kilometers of the Landfill from Waste Sources

	Waste Sources Locations	Distance (Km)
1	Township	2
2	Miruka	10
3	Kebirigo	10
4	Nyamaiya	17
5	Tinga	8

Table 5: Distance in Kilometers of the Recycling Facility from Waste Sources

	Waste Sources Locations	Waste Amount in Tones d_i
1	Township	5,475
2	Miruka	2,920
3	Kebirigo	2,555
4	Nyamaiya	1,460
5	Tinga	1,825

Table 6: The amount of Waste at the Sources

	Recycling Plant	Capacity
1	Township	$(\widetilde{2{,}000}, \widetilde{2{,}500}, \widetilde{3{,}000})$

Table 7: Recycling Plant's Capacity

	Landfill	$Q_{j=1}$ in Tones
1	Kemasare	$(\widetilde{13{,}500}, \widetilde{14{,}000}, \widetilde{14{,}500})$

Table 8: Landfill Location, Capacity

i/j	Source/Facility	Kemasare (j=1)	Township (j=2)
1	Township	$(\widetilde{1780}, \widetilde{1830}, \widetilde{1880})$	$(\widetilde{550}, \widetilde{600}, \widetilde{650})$
2	Miruka	$\widetilde{1720}, \widetilde{1770}, \widetilde{1820})$	$(\widetilde{700}, \widetilde{750}, \widetilde{800})$
3	Kebirigo	$(\widetilde{1940}, \widetilde{1990}, \widetilde{2040})$	$(\widetilde{700}, \widetilde{750}, \widetilde{800})$
4	Nyamaiya	$(\widetilde{1745}, \widetilde{1795}, \widetilde{1845})$	$(\widetilde{950}, \widetilde{1000}, \widetilde{1050})$
5	Tinga	$(\widetilde{2030}, \widetilde{2080}, \widetilde{2130})$	$(\widetilde{600}, \widetilde{650}, \widetilde{700})$

Table 9: Fuzzy Transportation Cost from Sources to Landfill (j=1) and Fuzzy + Recycling Facility (j=2) per Ton

	Operation Cost	Revenue
	$(\widetilde{1000}, \widetilde{1200}, \widetilde{1400})$	$(\widetilde{3600}, \widetilde{3800}, \widetilde{4000})$

Table 10: Fuzzy Operation Cost and Revenue at the Recycling Facility per Ton

Using the above information, the multi-objective problem of MSWM is formulated by taking X_{ij} and d_c as deterministic while the rest are TFN. Consequently,

$$Min\ Z_1 = (\widetilde{1780}, \widetilde{1830}, \widetilde{1880})x_{1,1} + (\widetilde{1720}, \widetilde{1770}, \widetilde{1820})x_{2,1} + (\widetilde{1940}, \widetilde{1990}, \widetilde{2040})x_{3,1} + (\widetilde{1745}, \widetilde{1795}, \widetilde{1845})x_{4,1} + (\widetilde{2030}, \widetilde{2080}, \widetilde{2130})x_{5,1} + (\widetilde{550}, \widetilde{600}, \widetilde{650})x_{1,2} + (\widetilde{700}, \widetilde{750}, \widetilde{800})x_{2,2} + (\widetilde{700}, \widetilde{750}, \widetilde{800})x_{3,2} + (\widetilde{950}, \widetilde{1000}, \widetilde{1050})x_{4,2} + (\widetilde{600}, \widetilde{650}, \widetilde{700})x_{5,2} + (\widetilde{1000}, \widetilde{1200}, \widetilde{1400})x_{i,2} \precsim g_1 \quad (39)$$

$$Max\ Z_2 = \sum_{i=1}^{n} (\widetilde{3600}, \widetilde{3800}, \widetilde{4000})x_{i2} \succsim g_2 \quad (40)$$

Therefore,

$\sum_{i=1}^{n}\sum_{j=1}^{m} X_{ij} = 14235, \quad i = 1,2..5, \; j = 1,2$ (41)

$\sum_{i=1}^{n} X_{i1} \leq (\widetilde{13,500}, \widetilde{14,000}, \widetilde{14,500})$ (42)

$\sum_{i=1}^{n} X_{i2} \leq (\widetilde{2,000}, \widetilde{2,500}, \widetilde{3,000})$ (43)

$X_{ij} \geq 0$

The decision maker assumes that X_{ij} and d_c are deterministic in the model and that the right-hand side variables are TFNs. If the decision-maker desires to spend Ksh. 25,977,050 to handle the amount of waste at the sources, he can only tolerate up to Ksh. 27,400,550. At the same time, the decision maker would like to collect Ksh. 10,000,000 from recycling waste but not less than Ksh 9,000,000. By applying deffuzification, the model is transformed to a deterministic form given by:

$Min\ Z_1 = 1830x_{1,1} + 1770x_{2,1} + 1990x_{3,1} + 1795x_{4,1} + 2080x_{5,1} + 1800x_{1,2} + 1950x_{2,2} + 1950x_{3,2} + 2200x_{4,2} + 1850x_{5,2} \precsim 25,977,050$ (44)

$Max\ Z_2 = \sum_{i=1}^{5} 3800x_{i,2} \succsim 10,000,000$ (45)

Such that:

$\sum_{i=1}^{n}\sum_{j=1}^{m} X_{ij} = 14235, \quad i = 1,2..5, \; j = 1,2$ (46)

$\sum_{i=1}^{n} X_{i1} \leq 14,000$ (47)

$\sum_{i=1}^{n} X_{i2} \leq 2,500$ (48)

$X_{ij} \geq 0$

By introducing deviations, the model becomes:

$Min\ Z_1 = 1830x_{1,1} + 1770x_{2,1} + 1990x_{3,1} + 1795x_{4,1} + 2080x_{5,1} + 1800x_{1,2} + 1950x_{2,2} + 1950x_{3,2} + 2200x_{4,2} + 1850x_{5,2} + d_1^- - d_1^+ = 25,977,050$ (49)

$Max\ Z_2 = \sum_{i=1}^{5} 3800x_{i,2} + d_2^- - d_2^+ = 10,000,000$ (50)

Such that equations from (46) to (48).

The model now minimizes the unwanted deviations:

Optimize $F = d_1^+ + d_2^-$ (51)

Such that,

$1830x_{1,1} + 1770x_{2,1} + 1990x_{3,1} + 1795x_{4,1} + 2080x_{5,1} + 1800x_{1,2} + 1950x_{2,2} + 1950x_{3,2} + 2200x_{4,2} + 1850x_{5,2} + d_1^- - d_1^+ = 25,977,050$ (52)

$\sum_{i=1}^{5} 3800x_{i,2} + d_2^- - d_2^+ = 10,000,000$ (53)

And other systems of equations from (46) to (48).

The model is then solved using the simplex method to obtain the values of deviations and, hence, obtain Z1 and Z_2.

Under these circumstances, the fuzzy- type of the linear membership functions for the objectives functions μZ_1 and μZ_2 is defined for the transportation and operation cost and revenue, respectively, as follows:

$$\mu Z_1 \cong \begin{cases} 0 & if \quad Z_1 \succsim 27,400,550 \\ \frac{27,400,550 - Z_1}{27,400,550 - 25,977,050} & if \quad 25,977,050 \precsim Z_1 \precsim 27,400,550 \\ 1 & if \quad Z_1 \precsim 25,977,050 \end{cases} \quad (54)$$

$$\mu Z_2 \cong \begin{cases} 0 & if \quad Z_2 \precsim 9,000,000 \\ \frac{Z_2 - 9,000,000}{10,000,000 - 9,000,000} & if \quad 9,000,000 \precsim Z_2 \precsim 10,000,000 \\ 1 & if \quad Z_2 \succsim 9,000,000 \end{cases} \quad (55)$$

4. Results and Discussion

Using the obtained data in the chapter three and the developed model, the parameters were substituted and using the proposed method the results were obtained. To obtain the optimal solution, a total of twenty-three iterations were performed.

From the Simplex Method processes illustrated above, the following results, shown in tables 11, 12 and 13, were obtained.

Model Variables	Solutions
$x_{1,1}$	4013
$x_{2,1}$	2769
$x_{3,1}$	2404
$x_{4,1}$	1365
$x_{5,1}$	1184
$x_{1,2}$	1462
$x_{2,2}$	151
$x_{3,2}$	151
$x_{4,2}$	95
$x_{5,2}$	641
d_1^-	0
d_1^+	5810075
d_2^-	500000
d_3^+	0
d_3^-	0
d_2^+	0
d_4^+	0
d_4^-	0

Table 11: Model Variables' Solutions

Deviations	Values
d_1^-	0
d_1^+	580075
d_2^-	500000
d_2^+	0

Table 12: Objective Functions' Deviational Values

Goal	Objective Function	Values
1	Z_1	26,557,125
2.	Z_2	9,500,000

Table 13: Objective Functions' Values

The obtained results illustrate that if the recycling facility is utilized to its full capacity, it will provide additional benefits from generated revenue. The remaining waste is taken to the landfill. The model assumes that the waste generated can be recycled for commercial benefits.

From the results above, the cost of managing 14,235 tons of waste is Ksh. 26,557,125, which falls below the optimal threshold of Ksh 27,400,550, as earlier projected. This operational cost consists of the overall costs of waste management from the source to the landfill, transportation costs from the source to the recycling facility, and the cost incurred at the recycling facility. This gives an MF of 0.600944068, which indicates a 60% satisfactory level.

The second objective function was intended to maximize the revenue from recycling 2,500 tons of waste. The results obtained show that Ksh 9,500,000 was generated from the recycling facility, which is the maximum revenue that can be generated from recycling. This indicates an MF value of 0.5, giving a 50% satisfactory level.

The results of this study are of immense practical utility since incorporating the recycling process in waste management will, in many ways, reduce the overall cost of waste management from a minimum of Ksh 25,977050 to Ksh 16,477,050, which is a 36% reduction.

5. Conclusion and Recommendations

5.1. Conclusion

Mathematical modelling has been found to be a very important tool in solving world problems. This study developed an FGP model to handle waste management in Nyamira municipality. The formulated model was solved using the simplex method linprog, a MATLAB software program. Sensitivity analysis was done on the recycling facility, which suggests that an increase in capacity will decrease the cost further. The study aimed to come up with a way in which the municipality can handle its waste at a lower cost. The study proposes an introduction of a recycling facility in the waste management system of Nyamira municipality; this will go a long way in reducing the cost of managing waste by about 36% by generating revenue. The output of the model reduces the amount of waste in the landfill, which will prolong its lifespan.

5.2. Recommendations

The element of time (dynamic element) needs to be introduced into the model; for instance, we can consider activities within time period $t = 1,2 \dots T_0$, where some parameters can change with time t. Planning involves time, and if an application is concerned with a situation that lasts for years, the same types of decisions may have to be made every year. When planning a multi-period horizon (say T_0), and there is no change in the data at all from one period to the next, then the optimum solution for the first period found from the static model for that period will remain optimal for each period in the planning horizon. In most multi-period problems, data changes from one period to the next are significant, and the optimum decisions for the various periods may be different, and the sequence of decisions will be interrelated. Designing a dynamic model with the aim of finding a sequence of decisions (one for every period) that is optimal for the planning horizon as a whole requires reasonably accurate estimates of data for every period of the planning horizon. This is a challenge, but if such data is available, a dynamic model tries to find the entire sequence of interrelated decisions that are optimal for the model over the entire planning horizon.

5.3. Suggestion for Future Research

The following areas can be considered for future research;

- More waste management alternatives can also be considered in the model, such as incinerators, compositing and waste-to-energy recovery,
- Conducting sensitivity analysis on the FGP model.

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