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On the Unit Groups of a Class of Total Quotient Rings of Characteristic p^k with $k \geq 3$

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Abstract

Let R be a commutative completely primary finite ring. The structures of the groups of units for certain classes of R have been determined. It is well known that completely primary finite rings play a crucial role in the endeavors towards the classification of finite rings. Let G be an arbitrary finite group. The classification of all finite rings R_i so that $U(R_i) \cong G$ is still an open problem. In this paper, we consider $S \subset R$ to be a saturated multiplicative subset of R and construct a total quotient ring R_S whose group of units is characterized, when char $R_S = p^k, k \geq 3$. It is observed that $U(R) \cong U(R_S)$, since $R \cong R_S$. The cases when char $R_S = p^k, k = 1, 2$ have been studied in a related work.

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1 Introduction

Let R be a commutative completely primary finite ring with identity 1. A subset $S \subset R$ is multiplicative if $1 \in S, 0 \notin S$ and $xy \in S$ if $x, y \in S$. Moreover, S is saturated if $y \notin S$ implies $x \notin S$ and $y \nmid x$. The set of equivalence classes $R \times S$ under the equivalence relation $(r, s) \sim (r', s')$ if and only if there exists $v \in S$ such that $v(rs' - r's) = 0$ shall be denoted by R_S or $S^{-1}R$. The equivalence class of (r, s) is denoted by $\left[\frac{r}{s}\right]$. Using addition and multiplication of quotients, it is easily verified that $S^{-1}R = \left\{\left[\frac{r}{s}\right] \mid r \in R, s \in S\right\}$ is a commutative ring with identity $\left[\frac{1}{1}\right]$. We may also regard $S^{-1}R$ as an $R-$algebra with the structure morphism $f : R \to S^{-1}R$ defined by $r \mapsto \left[\frac{r}{s}\right]$ verifiable as a ring homomorphism. The $R-$algebra structure on $S^{-1}R$ is defined by $r \cdot \left[\frac{r'}{s}\right] = \left[\frac{rr'}{s}\right]$.

Let R be a commutative ring with identity 1 and $S \subset R$ be a saturated multiplicative subset of R consisting of all the units, then $R_S = \left\{\left[\frac{r}{s}\right] \mid r \in R, s \in S\right\}$ endowed with usual addition and multiplication of fractions is a total quotient ring.

Throughout this paper, unless stated otherwise, $R_S = S^{-1}R$ denotes the total quotient ring, $U(R_S)$ denotes the unit groups of the total quotient rings R_S, $J(R_S)$ is the Jacobson radical of R_S, $GF(p^r) = R_S/J(R_S)$. The remaining notations are standard and can be obtained from the references e.g. [2] and [8].

The characterization of finite rings into well known structures is not complete. In particular, Ayoub[1] has studied the properties of finite primary rings and their groups of units, Chikunji [2, 3] has demonstrated an in depth study on the structure theory of completely primary finite rings. He has given the structures of the unit groups of certain classes of the said finite rings and a review of some other properties of the rings. Owino[7] has studied the units of commutative finite rings with zero divisors satisfying the idempotent property. It can be established from these findings that these unit groups are expressible as direct product of a cyclic group $\mathbb{Z}_{p^r-1}$ and the abelian $p-$group $1 + J(R)$ of order $p^{(k-1)r}$ where k, r are positive integers.

This paper therefore seeks to characterize $U(R_S)$ when char $p^k, k \geq 3$.

2 The Construction of the total quotient ring of characteristic p^k with $k \geq 3$

Let $R = R' \oplus U$ be an additive abelian group with identity, where $R' = GR(p^{kr}, p^k)$ is a Galois ring of order p^{kr} and characteristic p^k and U be a

$h-$dimensional $R'-$module. On this group, define multiplication by the following relation:

(i) When $k = 1, 2$ $pu_i = u_i u_j = u_j u_i = 0$, $u_i r' = (r')^{\sigma_i} u_i$

(ii) When $k \geq 3$, then $p^{k-1}u_i = p^2\gamma_{ij} = u_i^{k-1}u_j = 0$ for some $\gamma_{i,j} \in R^{'}$: $i \neq j$

where $r' \in R^{'}$, $1 \leq i, j \leq h$, p is a prime integer, k and r are positive integers and σ_i is the automorphism associated with u_i. Further, let the generators $\{u_i\}$ for U satisfy the additional condition that, if $u_i \in U$, then, $pu_i = u_i u_j = 0$. From the given multiplication in R, we see that if $r' + \sum_{i=1}^{h} \lambda_i u_i$ and $t' + \sum_{i=1}^{h} \gamma_i u_i$, $r', t' \in R'$, $\lambda_i, \gamma_i \in F'$ are elements of R, then,

$$(r' + \sum_{i=1}^{h} \lambda_i u_i)(t' + \sum_{i=1}^{h} \gamma_i u_i) = r't' + \sum_{i=1}^{h} \{(r' + pR')\gamma_i + \lambda_i (t' + pR')^{\sigma_i}\} u_i$$

It has been verified that the given multiplication turns the additive abelian group R, into a commutative ring with identity $(1, 0, ..., 0)$ or just 1 if and only if σ_i is the identity on R' (cf. see [6, 8]). Let $J(R)$ be its Jacobson radical and $S = U(R)$ be a multiplicatively closed subset of R. The localization of R with respect to S yields a total quotient ring R_S such that every element $[(r, s)] \in R_S$ is of the form $\left[(r^{'} + \sum_{i=1}^{h} \lambda_i u_i, \alpha + pr^{'} + \sum_{i=1}^{h} \lambda_i u_i)\right]$ where $\lambda_i \in \mathbb{F}_p', pr^{'} + \sum_{i=1}^{h} \lambda_i u_i \in J(R)$ and $\alpha \in S$. Let $[(r_1, s_1)], [(r_2, s_2)] \in R_S$. Define multiplication on R_S as follows:

$$[(r_1, s_1)][(r_2, s_2)] = [(r_1 r_2, s_1 s_2)] \qquad (*)$$

with the usual addition of fractions on R_S, the multiplication defined by $(*)$ turns R_S into a finite local ring with identity.

For any prime integer p and positive integers k, h and r the construction yields a total quotient ring in which $(J(R_S))^{k-1} \neq [(0, 1)]$ and $(J(R_S))^k = [(0, 1)]$. These rings are completely primary and therefore we can employ well known procedures to study their unit groups.

3 Preliminary Results

The following Theorems, Lemmata and Proposition shall be used in the sequel

Theorem 1. *(cf. [2]) Let G be a cyclic finite group such that $\mid G \mid = n$, then $G \cong \mathbb{Z}_n$.*

Theorem 2. *(cf. [5]) Let R be a finite ring. Then every left unit is a right unit and every left zero divisor is a right zero divisor. Furthermore, every element of R is either a zero divisor or a unit.*

Theorem 3. *(Property of localization of Completely primary rings) The only maximal ideals of $S^{-1}R$ are $S^{-1}J(R)$ where $J(R)$ is a maximal ideal of R such that $J(R) \cap S = \phi$*

Lemma 1. *(cf. [8]) Let R_S be a total quotient ring of the construction in section 2, then $U(R_S)$ is cyclic if and only if $\left[\frac{(1,0,\ldots,0)}{(1,0,\ldots,0)}\right] + J(R_S)$ is cyclic. Moreover*

$$U(R_S) = <\xi> \times \left[\frac{(1,0,\ldots,0)}{(1,0,\ldots,0)}\right] + J(R_S)$$

a direct product of the $p-$group $\left[\frac{(1,0,\ldots,0)}{(1,0,\ldots,0)}\right] + J(R_S)$ by the cyclic subgroup $<\xi>$ of order $p^r - 1$.

Lemma 2. *(cf. [8]) Let $\text{ann}(J(R_S))$ be the two sided annihilator of $J(R_S)$ then $\left[\frac{(1,0,\ldots,0)}{(1,0,\ldots,0)}\right] + ann(J(R_S))$ is a subgroup of $\left[\frac{(1,0,\ldots,0)}{(1,0,\ldots,0)}\right] + J(R_S)$*

Theorem 4. *Let R be a commutative ring with identity 1. Suppose $S \subseteq R$ is a multiplicative, then there exists a prime and maximal ideal P with respect to inclusion among all the ideals in the complement of S in R.*

Theorem 5. *Let R be a commutative ring with identity 1. Then the following are equivalent:*

i. *S is saturated*

ii. *$R - S = \cup_{i\in\Lambda} P_i$ where $\{P_i\} = Spec(R)$ such that $P_i \cap S = \phi$*

Theorem 6. *(Universal property of the Quotient Ring) The canonical homomorphism $f : R \to S^{-1}R$ defined by $f(r) = \left[\frac{r}{s}\right]$ is an $R-$algebra homomorphism, so that f satisfies the following axioms*

i. *f is a ring homomorphism and $f(s) \subseteq U(S^{-1}R)$.*

ii. *If $\theta : R \to R'$ is a ring homomorphism with $\theta(S) \subseteq U(R')$, then there exists a unique homomorphism $\psi : S^{-1}R \to R'$ such that $\psi \cdot f = \theta$.*

Theorem 7. *Let S be a multiplicative set of a unital commutative ring R. Then*

i. *Proper ideals of the ring $S^{-1}R$ are of the form $IS^{-1}R = S^{-1}I = \left\{\frac{i}{s} \mid i \in I, s \in S\right\}$ with $I \in R$ an ideal and $I \cap S = \phi$.*

ii. *Prime ideals in $S^{-1}R$ are of the form $S^{-1}P$ where P is prime in R and $P \cap S = \phi$.*

Theorem 8. *Let R be a commutative ring with identity and $S \subseteq R$ a multiplicative subset, I is an ideal of R and $S = Im(S)$, the natural image of S in R/I, then $S^{-1}R/S^{-1}I \cong S^{-1}(R/I)$ as an $R-$algebra.*

Remark 1. In order to completely classify the group of units of the rings constructed, we determine the structure of $\left[\frac{(1,0,\ldots,0)}{(1,0,\ldots,0)}\right] + J(R_S)$. Since $U(R_S)$ is abelian $\left[\frac{(1,0,\ldots,0)}{(1,0,\ldots,0)}\right] + J(R_S)$ is a normal subgroup of $U(R_S)$. Particularly, Let R_S be the total quotient ring of the construction section 2, then $\left[\frac{(1,0,\ldots,0)}{(1,0,\ldots,0)}\right] + J(R_S)$ is an abelian $p-$subgroup of the unit group $U(R_S)$. For brevity we shall in the sequel denote the identity $\left[\frac{(1,0,\ldots,0)}{(1,0,\ldots,0)}\right]$ as $\left[\frac{1}{1}\right]$.

4 Main results

Proposition 1. *(cf. [7]) Let R_S be the total quotient ring constructed above and $J(R_S)$ be it is Jacobson radical, then*

(i) $J(R_S) = \left\{\left[(pr' + \sum_{i=1}^{h} \lambda_i u_i, \alpha + pr' + \sum_{i=1}^{h} \lambda_i u_i)\right] \mid r' \in R', \lambda_i \in \mathbb{Z}_p, u_i \in R, \alpha \in U(R)\right\}$

(ii) $(J(R_S))^{k-1} = \left\{\left[(p^{k-1}r', \alpha + p^{k-1}r')\right] \mid r' \in R', \alpha \in U(R)\right\}$ *and*

(iii) $(J(R_S))^k = [(0, 1)]$

Let R_S be the total quotient ring of the construction in Section 2 with maximal ideal $J(R_S)$ such that $(J(R_S))^{k-1} \neq [(0, 1)]$ and $(J(R_S))^k = [(0, 1)]$. Then R_S is of order $p^{(k+h)r}$. Since R_S is of the given order and $U(R_S) = R_S - J(R_S)$, then $|U(R_S)| = p^{(k-1)r+rh}(p^r - 1)$ and $|\frac{1}{1} + J(R_S)| = p^{(k-1)r+rh}$. So $\frac{1}{1} + J(R_S)$ is an abelian p-group.

Proposition 2. *Let R_S be a local ring of characteristic p^k with $k \geq 3$. Then the group of units $U(R_S)$ of R_S contains a cyclic subgroup $< \xi >$ of order $p^r - 1$ and $U(R_S)$ is a semi direct product of $\left[\frac{1}{1}\right] + J(R_S)$ by $< \xi >$.*

In order to completely classify the groups of units of the total quotient ring, we determine the structure of $\left[\frac{1}{1}\right] + J(R_S)$.

We now give the generalization as follows;

Proposition 3. *The structure of the unit group $U(R_S)$ of the total quotient ring constructed in section 2 with characteristic $p^k, k \geq 3, r \geq 1$ and $h \geq 1$ is as follows:*

$$\left[\frac{1}{1}\right] + J(R_S) \cong \begin{cases} \mathbb{Z}_2 \times \mathbb{Z}_{2^{k-2}} \times \mathbb{Z}_{2^{k-1}}^{r-1} \times (\mathbb{Z}_2^r)^h & \text{if } p = 2; \\ \mathbb{Z}_{p^{k-1}}^r \times (\mathbb{Z}_p^r)^h & \text{if } p \neq 2. \end{cases}$$

and

$$U(R_S) \cong \begin{cases} \mathbb{Z}_{2^r-1} \times \mathbb{Z}_2 \times \mathbb{Z}_{2^{k-2}} \times \mathbb{Z}_{2^{k-1}}^{r-1} \times (\mathbb{Z}_2^r)^h & \textit{if } p = 2; \\ \mathbb{Z}_{p^r-1} \times \mathbb{Z}_{p^{k-1}}^r \times (\mathbb{Z}_p^r)^h & \textit{if } p \neq 2. \end{cases}$$

Proof. Let $\lambda_1, \ldots, \lambda_r \in R'$ with $\lambda_1 = 1$ such that $\overline{\lambda_1}, \ldots, \overline{\lambda_r} \in R'/pR'$ form a basis for R'/pR' regarded as a vector space over its prime subfield $\mathbb{F}_p$. Let $s_1, \ldots, s_h \in S = U(R)$. Now

$$U(R_S) = U\left(R_S/J(R_S)\right) \cdot \left(\left[\frac{1}{1}\right] + J(R_S)\right) \cong \mathbb{Z}_{p^r-1} \times \left(\left[\frac{1}{1}\right] + J(R_S)\right).$$

Since the two cases do not overlap, we consider them separately:
Let $p = 2$
The structures of $\left[\frac{1}{1}\right] + J(R_S)$. are obtained as follows;
Suppose $l = 1, \ldots, r, 1 \leq i \leq h$ and let $\psi \in R'$ such that $x^2 + x + \overline{\psi} = \overline{0}$ over R'/pR' has no solution in the field R'/pR' and $\overline{\psi} \in R'/pR'$, we obtain the following results:

$$\left(\left[\frac{1}{1} + \frac{2^{k-1}\lambda_1}{s}\right]\right)^2 = \left[\frac{1}{1}\right],$$

$$\left(\left[\frac{1}{1} + \frac{4\psi}{s}\right]\right)^{2^{k-2}} = \left[\frac{1}{1}\right].$$

Also

$$\left(\left[\frac{1}{1} + \frac{2\lambda_l}{s}\right]\right)^{2^{k-1}} = \left[\frac{1}{1}\right]$$

for $l = 2, \ldots, r$,

$$\left(\left[\frac{1}{1} + \frac{\lambda_l u_1}{s}\right]\right)^2 = \left[\frac{1}{1}\right], \left(\left[\frac{1}{1} + \frac{\lambda_l u_2}{s}\right]\right)^2 = \left[\frac{1}{1}\right], \ldots, \left(\left[\frac{1}{1} + \frac{\lambda_l u_h}{s}\right]\right)^2 = \left[\frac{1}{1}\right]$$

for every $l = 1, \ldots, r$. Now, consider positive integers $\alpha, \beta, \kappa_l, \tau_{1l}, \ldots, \tau_{hl}$ with $\alpha \leq 2, \beta \leq 2^{k-2}, \kappa_l \leq 2^{k-1}, \tau_{il} \leq p (1 \leq i \leq h, 1 \leq l \leq r)$, we notice that the equation

$$\left(\left[\frac{1}{1} + \frac{2^{k-1}\lambda_1}{s}\right]\right)^{\alpha} \cdot \left(\left[\frac{1}{1} + \frac{4\psi}{s}\right]\right)^{\beta} \cdot \prod_{l=2}^{r} \left\{\left(\left[\frac{1}{1} + \frac{2\lambda_l}{s}\right]\right)^{\kappa_l}\right\} \cdot$$

$$\prod_{l=1}^{r} \left\{\left(\left[\frac{1}{1} + \frac{\lambda_l u_1}{s}\right]\right)^{\tau_{1l}}\right\} \cdot \prod_{l=1}^{r} \left\{\left(\left[\frac{1}{1} + \frac{\lambda_l u_2}{s}\right]\right)^{\tau_{2l}}\right\} \cdot$$

$$\cdots \prod_{l=1}^{r} \left\{\left(\left[\frac{1}{1} + \frac{\lambda_l u_h}{s}\right]\right)^{\tau_{hl}}\right\} = \left\{\left[\frac{1}{1}\right]\right\}$$

will imply $\alpha = 2, \beta = 2^{k-2}, \kappa_l \leq 2^{k-1}$ for $l = 2, \ldots, r$ and $\tau_{il} = 2$ for every $l = 1, \ldots, r$ and $1 \leq i \leq h$.
If we set

$$H = \left\{ \left(\left[\frac{1}{1} + \frac{2^{k-1}\lambda_1}{s} \right] \right)^{\alpha} \mid \alpha = 1, 2 \right\}$$

$$G = \left\{ \left(\left[\frac{1}{1} + \frac{4\psi}{s} \right] \right)^{\alpha} \mid \beta = 1, \ldots, 2^{k-2} \right\}$$

$$T_l = \left\{ \left(\left[\frac{1}{1} + \frac{2\lambda_l}{s} \right] \right)^{\kappa} \mid \kappa = 1, \ldots, 2^{k-1} \right\}; l = 2, \ldots, r$$

$$S_{1l} = \left\{ \left(\left[\frac{1}{1} + \frac{\lambda_l u_1}{s} \right] \right)^{\tau_1} \mid \tau_1 = 1, 2 \right\}$$

$$S_{2l} = \left\{ \left(\left[\frac{1}{1} + \frac{\lambda_l u_2}{s} \right] \right)^{\tau_2} \mid \tau_2 = 1, 2 \right\}$$

$$\vdots$$

$$S_{hl} = \left\{ \left(\left[\frac{1}{1} + \frac{\lambda_l u_h}{s} \right] \right)^{\tau_h} \mid \tau_h = 1, 2 \right\};$$

it is easily noticed that $H, G, T_l, S_{1l}, S_{2l}, \ldots, S_{hl}$ are cyclic subgroups of the group $\frac{1}{1} + J(R_S)$ and they are of the orders indicated in their definition. Since,

$$|< \left[\frac{1}{1} + \frac{2^{k-1}\lambda_1}{s} \right] >| \cdot |< \left[\frac{1}{1} + \frac{4\psi}{s} \right] >| \cdot \prod_{l=2}^{r} | \cdot < \left[\frac{1}{1} + \frac{2\lambda_l}{s} \right] >|$$

$$\cdot \prod_{l=1}^{r} | \cdot < \left[\frac{1}{1} + \frac{\lambda_l u_1}{s} \right] >| \cdot \prod_{l=1}^{r} |< \left[\frac{1}{1} + \frac{\lambda_l u_2}{s} \right] >| \cdot \ldots$$

$$\cdot \prod_{l=1}^{r} |< \left[\frac{1}{1} + \frac{\lambda_l u_h}{s} \right] >| = p^{(h+k-1)r},$$

the intersection of any pair of the cyclic subgroups yields the identity group $\left[\frac{1}{1}\right]$, and the product of the $h+3$ subgroups $H, G, T_l, S_{1l}, S_{2l}, \ldots, S_{hl}$ is direct and exhausts the group $\frac{1}{1} + J(R_S)$.

Let $p \neq 2$
For $l = 1, \ldots, r$,

$$\left(\left[\frac{1}{1} + p\lambda_l \right] \right)^{p^{k-1}} = \left[\frac{1}{1} \right], \left(\left[\frac{1}{1} + \frac{\lambda_1 u_1}{s} \right] \right)^{p} = \left[\frac{1}{1} \right], \ldots, \left(\left[\frac{1}{1} + \frac{\lambda_h u_h}{s} \right] \right)^{p} = \left[\frac{1}{1} \right].$$

For positive integers $\alpha_l, \beta_{1l}, \ldots, \beta_{hl}$ with $\alpha_l \leq p^{k-1}, \beta_{il} \leq p (1 \leq i \leq h)$, we notice that the equation

$$\prod_{l=1}^{r}\left\{\left(\begin{bmatrix}\frac{1}{1}+p\lambda_l\end{bmatrix}\right)^{\alpha_l}\right\}\cdot\prod_{l=1}^{r}\left\{\left(\begin{bmatrix}\frac{1}{1}+\frac{\lambda_1 u_1}{s}\end{bmatrix}\right)^{\beta_{1l}}\right\}\cdot$$

$$\cdots\cdot\prod_{l=1}^{r}\left\{\left(\begin{bmatrix}\frac{1}{1}+\frac{\lambda_h u_h}{s}\end{bmatrix}\right)^{\beta_{hl}}\right\}=\left\{\begin{bmatrix}\frac{1}{1}\end{bmatrix}\right\}$$

will imply $\alpha_l = p^{k-1}, \beta_{il} = p (1 \leq i \leq h, l = 1, \ldots, r)$. If we set

$$T_l = \left\{\left(\begin{bmatrix}\frac{1}{1}+p\lambda_l\end{bmatrix}\right)^{\alpha} \mid \alpha = 1, \ldots, p^{k-1}\right\}$$

$$S_{1l} = \left\{\left(\begin{bmatrix}\frac{1}{1}+\frac{\lambda_1 u_1}{s}\end{bmatrix}\right)^{\alpha} \mid \alpha = 1, \ldots, p^{k-1}\right\}$$

$$\vdots$$

$$S_{hl} = \left\{\left(\begin{bmatrix}\frac{1}{1}+\frac{\lambda_h u_h}{s}\end{bmatrix}\right)^{\alpha} \mid \alpha = 1, \ldots, p^{k-1}\right\}$$

We see that $T_l, S_{1l}, \ldots, S_{hl}$ are all cyclic subgroups of $\left[\frac{1}{1}\right] + J(R_S)$ and they are of the orders indicated by their definitions. Since

$$\prod_{l=1}^{r}|<\begin{bmatrix}\frac{1}{1}+p\lambda_l\end{bmatrix}>|\cdot\prod_{l=1}^{r}|<\begin{bmatrix}\frac{1}{1}+\frac{\lambda_1 u_1}{s}\end{bmatrix}>|\cdots\cdot\prod_{l=1}^{r}|<\begin{bmatrix}\frac{1}{1}+\frac{\lambda_h u_h}{s}\end{bmatrix}>|=p^{(h+k-1)r}$$

and the intersection of any pair of the cyclic subgroups gives the identity group, the product of the $(h+1)r$ subgroups $T_l, S_{1l}, \ldots, S_{hl}$ is direct and the product exhausts the group $\left[\frac{1}{1}\right] + J(R_S)$. □

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